Sr. No. of Question Paper:

Unique Paper Code: 12277504

Name of the Paper: Topics in Microeconomics - I

Name of the Course: CBCS DSE

Semester: V

Maximum Marks: 75

Instructions for Candidates

There are *six* questions in all. Answer any *four* questions. All questions carry equal marks.

1. Consider the demand function

$$q_i = 2 - 2p_i + p_j$$

$i = 1,2$, $i \neq j, j$ is the other firm, q_i is quantity and p_i the price charged by firm i.

Assume that the firms move sequentially and choose their prices with firm 1 moving first followed by firm 2. Both firms want to maximize their profits and both firms' marginal costs are zero.

(a) Set this up as an extensive game with perfect information.
(b) Solve for the subgame perfect equilibria and also find the profits of both the firms.
(c) How is the subgame perfect outcome above different from the outcome that would have resulted had the firms moved simultaneously instead of sequentially? Does the firm moving first enjoy a higher level of profits than the firm moving second in the sequential move game? Compare these profit levels to the profit levels of both firms when they take the decisions simultaneously.

2. Suppose that the inverse domestic demand for some commodity is $P(Q) = 15 - Q$, where Q is total quantity and P the price in dollars. Suppose that there are two firms supplying in this market, a domestic firm and a foreign firm. Imagine that these firms possess identical technologies and that transportation costs for the foreign firm are negligible. Specifically assume that the cost function of each firm is

$$C_D(q_D) = 3q_D \text{ and } C_F(q_F) = 3q_F$$

where the subscripts stand for "domestic" and "foreign" and $Q = q_D + q_F$

(i) Suppose that these firms engage in a one period quantity competition (a Cournot game). Derive the best response functions and solve for the Cournot-Nash equilibrium. What is the price under the Cournot outcome?
(ii) Suppose that the government in the foreign firm's country(from here on 'foreign government') decides to subsidize exports by paying the foreign firm a subsidy of s dollars for every unit it sells abroad. Suppose also that the size of the subsidy is known to both firms before they choose their respective output levels. Derive the best response functions of both the firms and solve for the Cournot-Nash equilibrium output levels for each firm. What is the market price? (Note: These will be functions of s).
(iii) Suppose the objective of the foreign government is to maximize the profit of the foreign firm net of subsidy payments. What is the value of the optimal subsidy s*?

3. Consumers are uniformly distributed along a boardwalk that is 1 kilometer long. Ice-cream prices are regulated, so consumers go to the nearest vendor because they dislike walking (assume that at the regulated prices all consumers will purchase an ice cream even if they have to walk a kilometer). If more than one vendor is at the same location, they split the business evenly.

(i) Consider a game in which two ice-cream vendors pick their locations simultaneously. Model this situation as a strategic game. In particular, give the exact values of the payoff functions of the two vendors as a function of their relative locations.

(ii) Show that there exists a unique pure strategy Nash equilibrium and that it involves both vendors locating at the midpoint of the boardwalk.
(iii) Show that with three vendors, no pure strategy Nash equilibrium exists.

4. (a) Suppose you play in a football team, and you are about to take a penalty kick. You have to decide whether to kick to the left or right corner of the goal. Your opponent team's goalkeeper, in turn, has to decide whether to dive left or right. To put some numbers to this, assume that if the goalkeeper dives left (right) when you kick left (right), then the goalkeeper blocks the kick with probability one. On the other hand, if you kick left (right) and the goalkeeper dives right (left), then you will definitely score a goal with probability one.

(i) Model this story as a strategic game with ordinal preferences (use a matrix in which the payoffs of the penalty kicker and the goalkeeper are the probabilities of scoring a goal and blocking a kick, respectively, for any combination of actions).
(ii) Find all the pure strategy and mixed strategy Nash equilibrium/equilibria of this game.
(iii) Find all the Nash equilibria of the game when the penalty kicker has 2/3 chance of scoring if he kicks left and the goalkeeper dives left, and only 1/3 chance if he kicks right and the goalkeeper dives right.

(b) A child's action a affects both her own private income $c(a)$ and her parent's income $p(a)$; for all values of a we have $c(a) < p(a)$. The child is selfish: she cares only about the amount of money she has. Her loving parent cares both about how much money she has and how much her child has. Specifically, the preferences of the parent are represented by a payoff function U_P equals to the smaller of the amount of money she has and the amount of money her child has.
The parent may transfer money to the child. This transfer is denoted by $t \geq 0$. In that case, the utility U_C of the child is given by:

$$U_C(a, t) = c(a) + t$$

whereas the utility of the parent is:

$$U_P(a, t) = min\{p(a) - t, c(a) + t\}$$

The timing is as follows. First the child takes an action, then the parent decides how much money to transfer.

(i) Model this situation as an extensive game.
(ii) Show that in a subgame perfect equilibrium the child takes an action that maximizes the sum of her private income and the parent's income.
(iii) Assume that $c(a) = a$ and $p(a) = 2a$ where $a \in [0,1]$. Find the unique subgame perfect equilibrium of this game. Also find the equilibrium utilities of the parent and the child.

5. (a) Two players find themselves in a legal battle over a patent. The patent is worth 20 to each player, so the winner would receive 20 and the losing player 0. Given the norms of the country, it is common to bribe the judge hearing a case. Each player can offer a bribe secretly, and the one whose bribe is the highest will be awarded the patent. If both choose not to bribe, or if the bribes are the same amount, then each has an equal chance of being awarded the patent. If a player does bribe,

then the bribe can be either valued at either 9 or 20. (Assume that any other number is unlucky and the judge would surely rule against a player offering such a bribe).

(i) Find the pure strategy Nash equilibrium for this game.
(ii) Consider that the norms are different now and that a bribe of 15 is also acceptable. Find all symmetric Nash equilibria of this game. (Look for both pure strategy and mixed strategy Nash equilibria. The actions are now 0 (no bribe), 9, 15 and 20.

(b) In the Envelope Game, there are two players and two envelopes. One of the envelope is marked "player 1" and the other "player 2." At the beginning of the game, each envelope contains one dollar. Player 1 is given the choice between stopping the game and continuing. If he chooses to stop, then each player gets the money in his own envelope and the game ends. If player 1 chooses to continue, then a dollar is removed from his envelope and two dollars are added to player 2's envelope. Then player 2 must choose between stopping the game and continuing. If he stops, then the game ends and each player keeps the money in his one envelope. If player 2 chooses to continue, then a dollar is removed from his envelope and two dollars are added to player 1's envelope. Play continues like this alternating between the players, until either one of them decides to stop or k rounds have elapsed. If neither player decides to stop by the end of the kth round, then both players obtain zero. Assume that both players want to maximize the amount of money they earn.

(i) Draw a game-tree for $k = 5$.
(ii) Find the subgame perfect equilibrium when $k = 5$

6. (a) Three roommates need to vote on whether they will adopt a new rule and clean their apartment once a week or stick to the current once a month rule. Each votes "yes" for the new rule or "no" (in favor of the current rule). Players 1 and 2 prefer the new rule while player 3 prefers the old rule.

(i) Imagine that the players require a unanimous vote to adopt the new rule. Player 1 votes first, then player 2, and then player 3, the latter two observing the previous votes. Model this as an extensive game with perfect information. Find the Nash equilibrium/equilibria of this game. Also find the subgame perfect equilibrium/equilibria.
(ii) Imagine now that the players require a majority rule to implement the new rule. (At least two "yes" votes). The order of moves is the same as before. Draw the game tree. Find all Nash equilibrium/ equilibria. Also find the subgame perfect equilibrium/equilibria.

(b) Three firms are considering entering a new market. The payoff for each firm that enters is $\frac{150}{n}$, where n is the number of firms that enter. The cost of entering is 62.

(i) Find all pure strategy Nash equilibria.
(ii) Are there any pure strategy Nash equilibria that is symmetric? If yes, give the same.
(iii) Find the symmetric mixed strategy Nash equilibrium/equilibria.