CC-V: Mathematical Physics-II (32221301)
Credit : 06 (Theory-04, Practical-02)
Theory : 60 Hours
Practical : 60 Hours

Course Objective

The emphasis of course is to equip students with the mathematical tools required in solving problems interest to physicists and expose them to fundamental computational physics skills thus enabling them to solve a wide range of physics problems. This course will aim at introducing the concepts of Fourier series, special functions, linear partial differential equations by separation of variable method.

Course Learning Outcomes

On successfully completing this course, the students will be able to

- Represent a periodic function by a sum of harmonics using Fourier series and their applications in physical problems such as vibrating strings etc.
- Obtain power series solution of differential equation of second order with variable coefficient using Frobenius method.
- Understand properties and applications of special functions like Legendre polynomials, Bessel functions and their differential equations and apply these to various physical problems such as in quantum mechanics.
- Learn about gamma and beta functions and their applications.
- Solve linear partial differential equations of second order with separation of variable method.
- In the laboratory course, the students will learn the basics of the Scilab software/Python interpreter and apply appropriate numerical method to solve selected physics problems both using user defined and inbuilt functions from Scilab/Python. They will also learn to generate and plot Legendre polynomials and Bessel functions and verify their recurrence relation.

Unit 1

Fourier Series: Periodic functions. Orthogonality of sine and cosine functions, Dirichlet Conditions (Statement only). Expansion of periodic functions in a series of sine and cosine functions and determination of Fourier coefficients. Even and odd functions and their Fourier expansions (Fourier Cosine Series and Fourier Sine Series). Application. Summing of Infinite Series. Parseval's Identity and its application to summation of infinite series.

(17 Lectures)

Unit 2

Frobenius Method and Special Functions: Singular Points of Second Order Linear Differential Equations and their importance. Frobenius method and its applications to differential equations: Legendre, Bessel, Hermite and Laguerre Differential Equations. Properties of Legendre Polynomials: Rodrigues Formula, Generating Function, Orthogonality. Simple recurrence relations. Expansion of function in a series of Legendre Polynomials. Bessel Functions of the First Kind: Generating Function, simple recurrence relations. Zeros of Bessel Functions ($J_o(x)$ and $J_1(x)$) and Orthogonality.

(24 Lectures)

Unit 3

Some Special Integrals: Beta and Gamma Functions and Relation between them. Expression of Integrals in terms of Gamma Functions.

(4 Lectures)

Unit 4

Partial Differential Equations: Solutions to partial differential equations (2 or 3 independent variables) using separation of variables: Laplace's Equation in problems of rectangular geometry. Solution of wave equation for vibrational modes of a stretched string, rectangular and circular membranes. Solution of 1D heat flow equation. (Wave/Heat equation not to be derived).

(15 Lectures)

Practical : 60 Hours

The aim of this Lab is to use the computational methods to solve physical problems. The course will consist of practical sessions and lectures on the related theoretical aspects. The recommended group size for the lab is not more than 15 students. Evaluation done not only on the basis of programming but also on the basis of formulating the problem. Minimum 12 programs must be attempted taking at least one from each programming section. The instructor may choose to use Python in place of Scilab covering all features as mentioned.

Topics	Description with Applications
Introduction to Numerical computation software using Scilab or Python	Introduction to Scilab, Advantages and disadvantages, Scilab environment, Command window, Figure window, Edit window, Variables and arrays, Initializing variables in Scilab, Multidimensional arrays, Sub-array, Special values, Displaying output data, data file, Scalar and array operations, Hierarchy of operations, Built in Scilab functions, Introduction to plotting, 2D and 3D plotting, Branching Statements and program design, Relational and logical operators, the while loop, for loop, details of loop operations, break and continue statements, nested loops, logical arrays and vectorization. User defined functions, Introduction to Scilab functions, Variable passing in Scilab, optional arguments, preserving data between calls to a function, Complex and Character data, string function, Multidimensional arrays an introduction to Scilab file processing, file opening and closing, Binary I/o functions, comparing binary and formatted functions, Numerical methods and developing the skills of writing a program.
Interpolation by Newton Gregory Forward and Backward difference formula, Error estimation of linear interpolation. Lagrange Interpolation.	Evaluation of trigonometric functions e.g. *sin(x), cos(x), tan(x)* etc – Given the values at n points in a tabulated form, evaluate the value at an intermediate point.
Numerical Integration: Newton Cotes Integration methods (Trapezoidal and Simpson rules) for definite integrals	Given acceleration with equidistant time data calculate position and velocity and plot them. Application to other mathematical and physical problems
Solution of Linear system of equations: Solve system of linear equations using Gauss elimination method and Gauss Seidal method. Inverse of a matrix (by Gauss elimination)	Application to Solution of mesh equations of electric circuits (3 meshes) Solution of coupled spring mass systems (3 masses)

Generation of Special functions using user defined functions and compare with Scilab built in functions	Generating and plotting Legendre Polynomials Generating and plotting Bessel functions Verification of recurrence relation Use the data obtained above for Legendre polynomials or Bessel's function at N points and find its value at an intermediate point using Lagrange interpolation.
Solution of Ordinary Differential Equations (ODE) First order Differential equation Euler, modified Euler and Runge-Kutta (RK) second and fourth order methods	First order differential equation (Initial value problems) Radioactive decay Current in RC, LC circuits with DC source Newton's law of cooling Classical equations of motion
System of First order Differential Equations	Attempt following problems using RK 4 order method: • Solve the coupled differential equations $dx/dt=y+x-x^3/3$; $dy/dt= -x$ for four initial conditions : $x(0) = 0$, $y(0) = -1, -2, -3, -4$. Plot x vs y for each of the four initial conditions on the same screen for $0 \le t \le 15$ • Application to linear electric networks

Second order differential equation (Euler and RK Methods)	Second Order Differential Equations: Harmonic oscillator (no friction) Damped Harmonic oscillator (Overdamped, Critically damped and Oscillatory behavior) Forced Harmonic oscillator (Transient and Steady state solution) Apply above to LCR circuits also The differential equation describing the motion of a pendulum is $\frac{d^2\theta}{dt^2} = -\sin\theta$. The pendulum is released from rest at an angular displacement α, i.e. $\theta(0) = \alpha$ and $\theta'(0) = 0$. Solve the equation for α = 0.1, 0.5 and 1.0 and plot $\theta, \frac{d\theta}{dt}$ as a function of time in the range $0 \leqslant t \leqslant 8\pi$. Also plot the analytic solution valid for small $\theta(\sin\theta \approx \theta)$ Solve $$x^2\frac{d^2y}{dx^2} - 4x(1+x)\frac{dy}{dx} + 2(1+x)y = x^3$$ with the initial conditions at x = 1 as $y(1) = \frac{1}{2}e^2, \frac{dy}{dx}(x=1) = \frac{-3}{2}e^2 - 0.5,$ in the range $1 < x < 3$. Plot y and $\frac{dy}{dx}$ against x in the given range on the same graph.
Using Scicos/xcos	Generating sine wave, square wave, sawtooth wave Solution of harmonic oscillator Phase space plots

References for Theory:

Essential Readings :

1. Advanced Engineering Mathematics, Erwin Kreyszig, 2008, Wiley India .
2. Advanced Mathematics for Engineers and Scientists: Schaum Outline Series, M. R Spiegel, McGraw Hill Education (2009).
3. Dfferential Equations, George F. Simmons, 2006, Tata McGraw-Hill.
4. Mathematical Methods for Physicists, Arfken, Weber and Harris, Elsevier
5. Applied Mathematics for Engineers and Physicists, L.A. Pipes and L.R. Harvill, Dover Publications (2014).

Additional Readings:

1. Mathematical methods for Scientists & Engineers, D.A.Mc Quarrie, 2003, Viva Books
2. Mathematical Methods for Physics and Engineers, K.F Riley, M.P. Hobson and S. J. Bence, 3rd ed., 2006, Cambridge University Press
3. Mathematical Physics, A.K. Ghatak, I.C. Goyal and S.J. Chua, Laxmi Publications Private Limited (2017)
4. Partial Differential Equations for Scientists and Engineers, S.J. Farlow, Dover Publications (1993).
5. Fourier Analysis with Applications to Boundary Value Problems: Schaum Outline Series, M. R Spiegel, McGraw Hill Education (1974).

References for Laboratory Work:

1. Simulation of ODE/PDE Models with MATLAB®, OCTAVE and SCILAB: Scientific and Engineering Applications: A. Vande Wouwer, P. Saucez, C. V. Fernández. 2014 Springer ISBN: 978-3319067896.
2. Documentation at the Scilab homepage: https://www.scilab.org/ and the Python home page https://docs.python.org/3/
3. Computational Physics, Darren Walker, Scientific International Pvt. Ltd (2015).
4. Applied numerical analysis, Cutis F. Gerald and P.O. Wheatley, Pearson Education, India (2007).
5. An Introduction to Computational Physics, T. Pang, Cambridge University Press (2010).

CC-VI: Thermal Physics (32221302)
Credit : 06 (Theory-04, Practical-02)
Theory : 60 Hours
Practical : 60 Hours

Course Objective

This course deals with the relationship between the macroscopic properties of physical systems in equilibrium. It reviews the concepts of thermodynamics learnt at school from a more advanced perspective and develops them further. The primary goal is to understand the fundamental laws of thermodynamics and their applications to various systems and processes. In addition, it will also give exposure to students about the Kinetic theory of gases, transport phenomena involved in ideal gases, phase transitions and behavior of real gases.